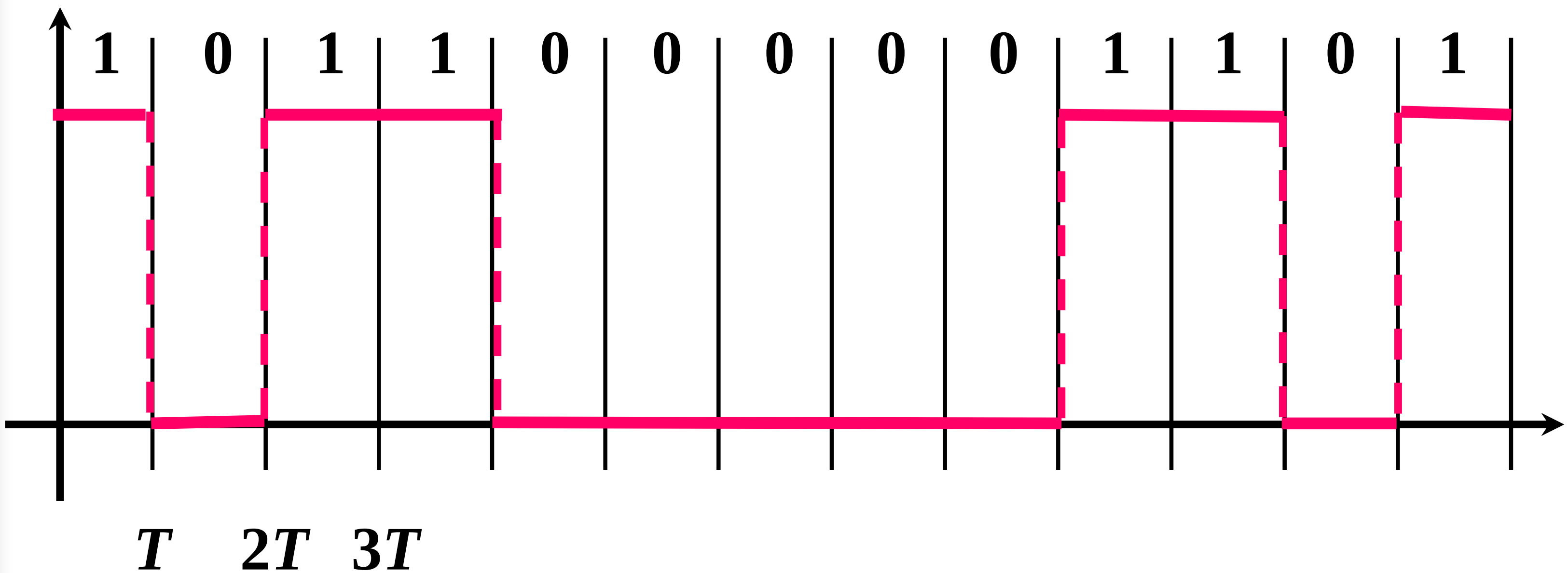
INTERSYMBOL INTERFERENCE (13)

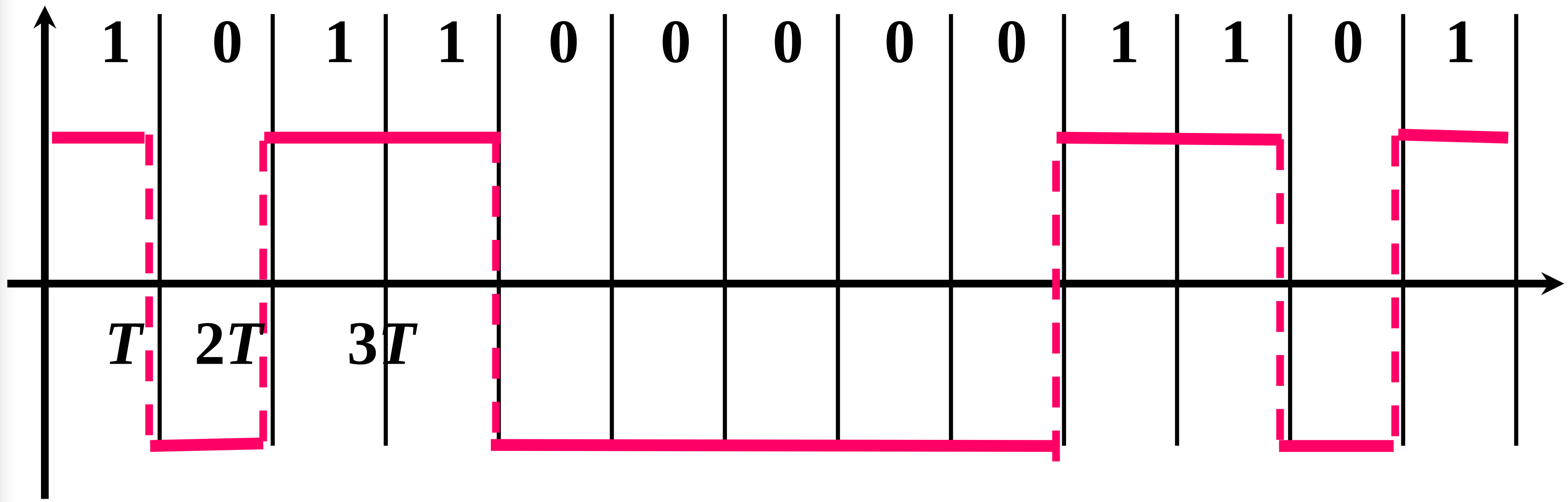
Binary data (logical symbols)



Unipolar transmission (line) code (*T* – clock)
(NRZ – Non Return to Zero)

Transmission (line) code is a mapping of binary (logical) symbols to electrical symbols.

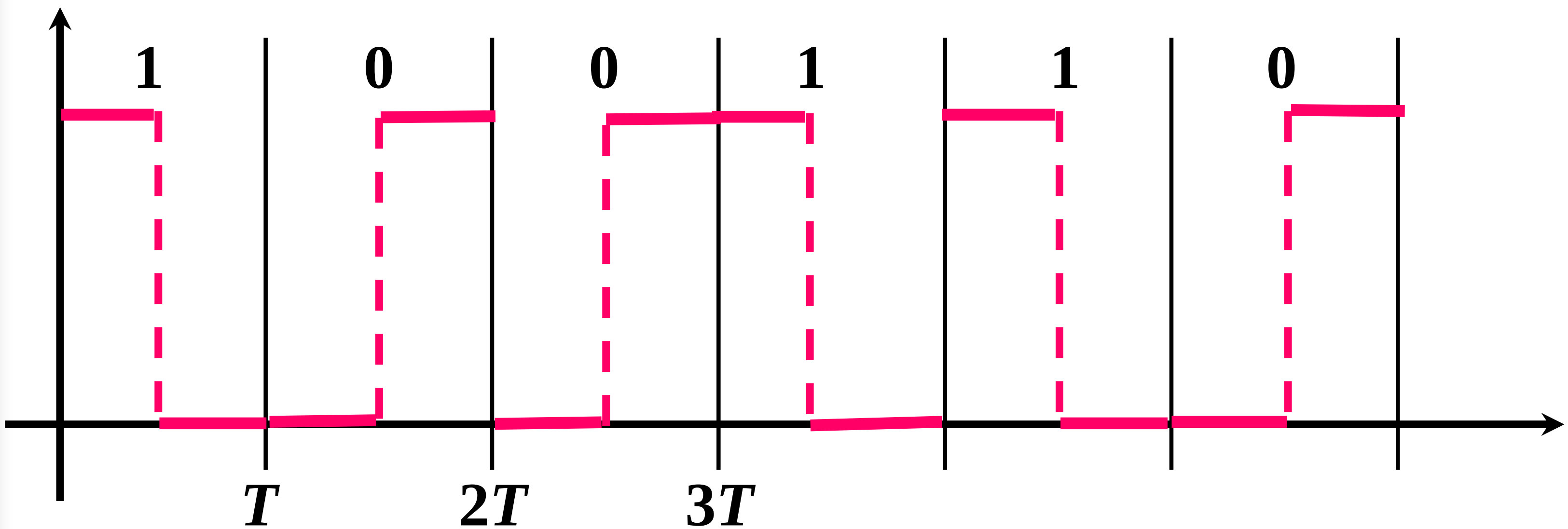
Bipolar transmission code



Bipolar transmission (line) code (*T* – clock)
(NRZ – Non Return to Zero)

Transmission (line) code is a mapping of binary (logical) symbols to electrical symbols.

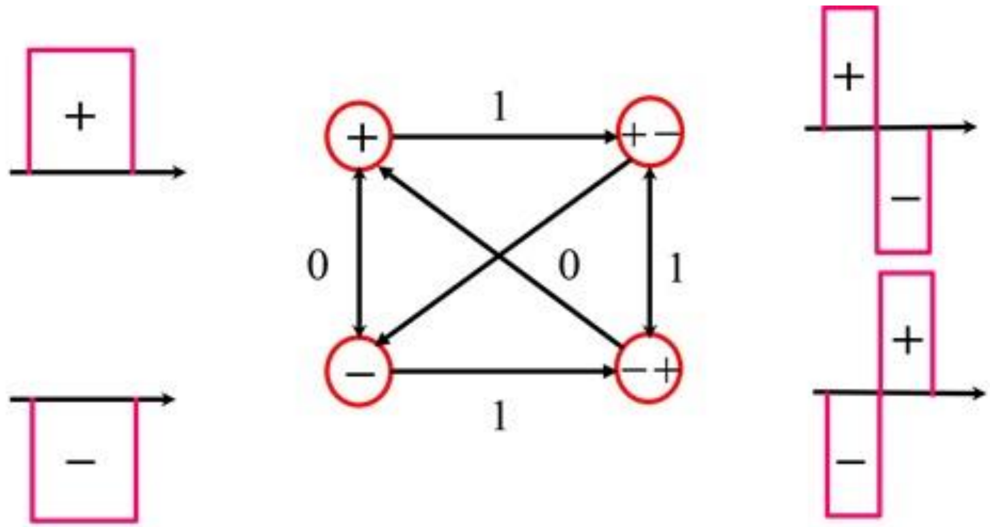
Biphase transmission code



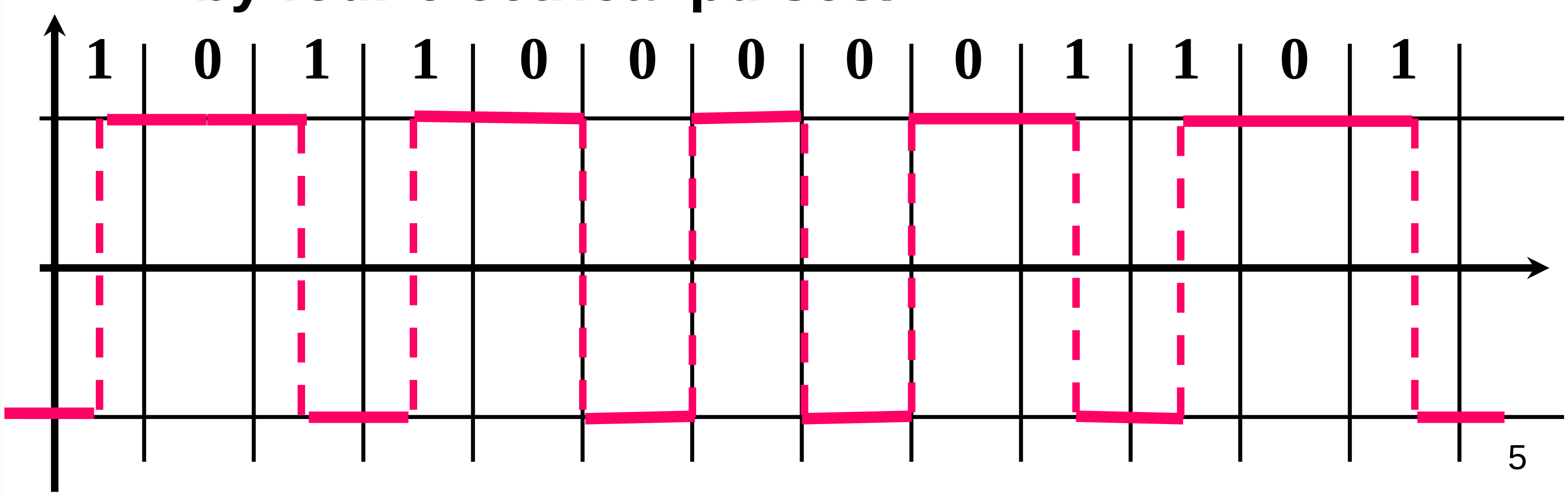
Biphase transmission (line) code (RZ – Return to Zero)

Logical symbols are represented by inversed biphase pulses.

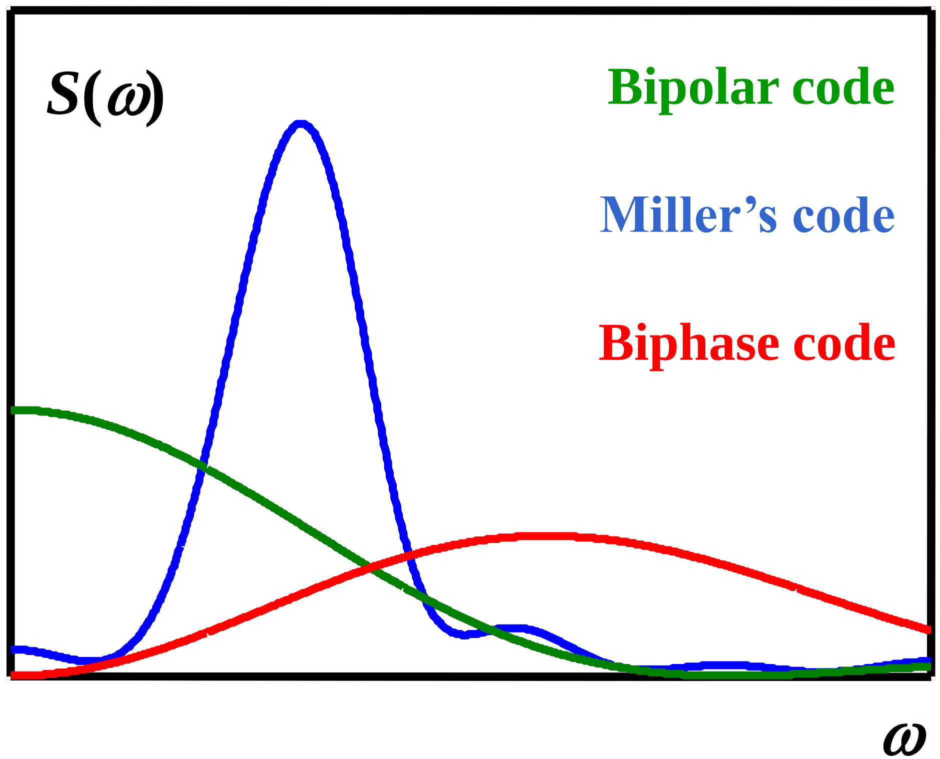
Miller's transmission code



Transition graph for the Miller's code
Two logical symbols are represented by four electrical pulses.

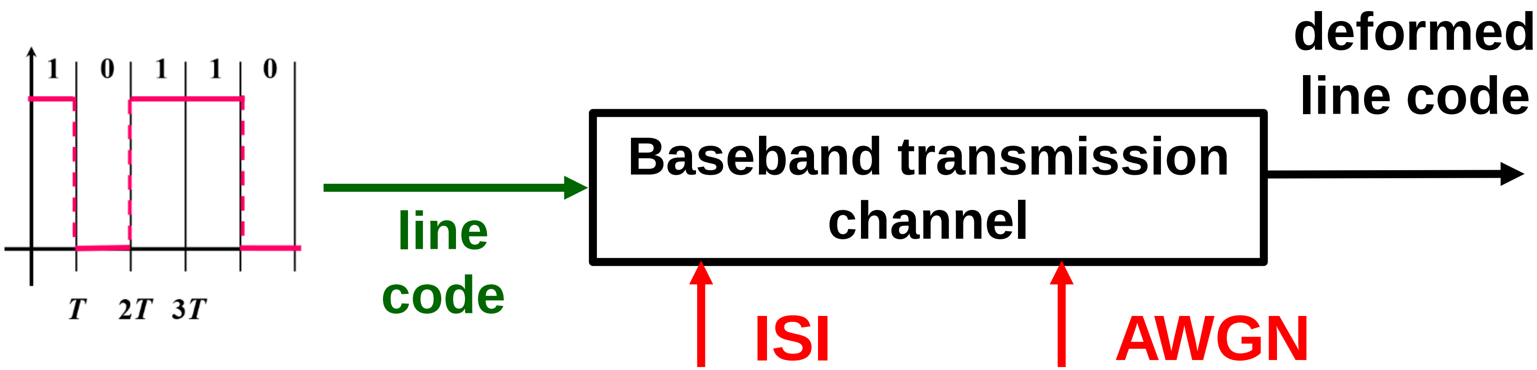


Transmission code power spectra



Shape of a spectral density function determines numerous features of a transmission code.

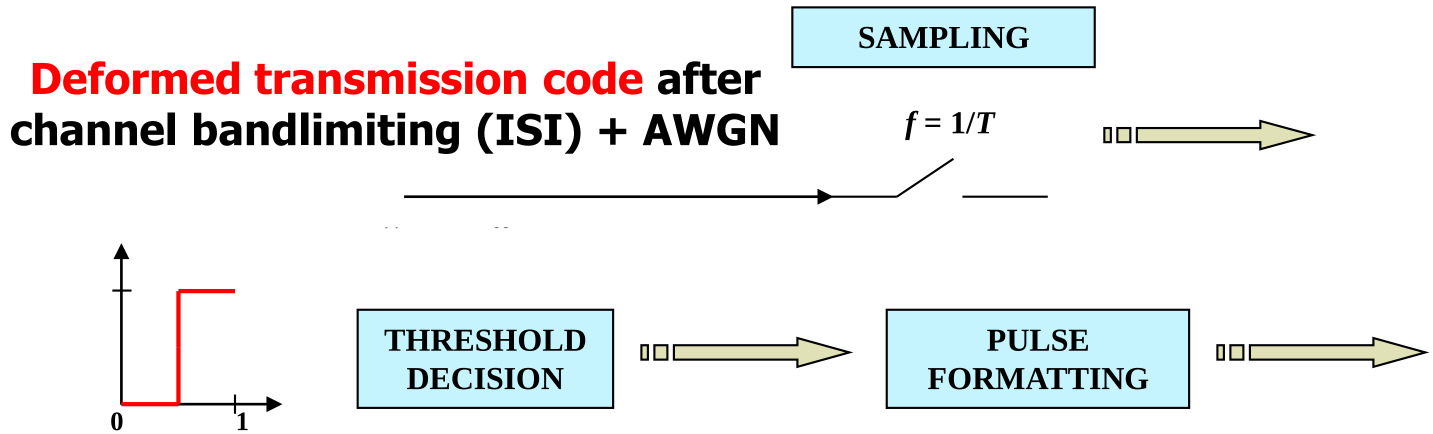
Deformation of line code and countermeasures



Deformation of line code and countermeasures:

- **AWGN interfers with line code**
  (decision based approach to detection of symbols of a line code)
- **Limited bandwidth of a baseband transmission channel causes a nonzero intersymbol interference (ISI $\neq 0$)**
  (transmission conformed with the Nyquist criterion (ISI = 0)

Line code receiver – basic concept



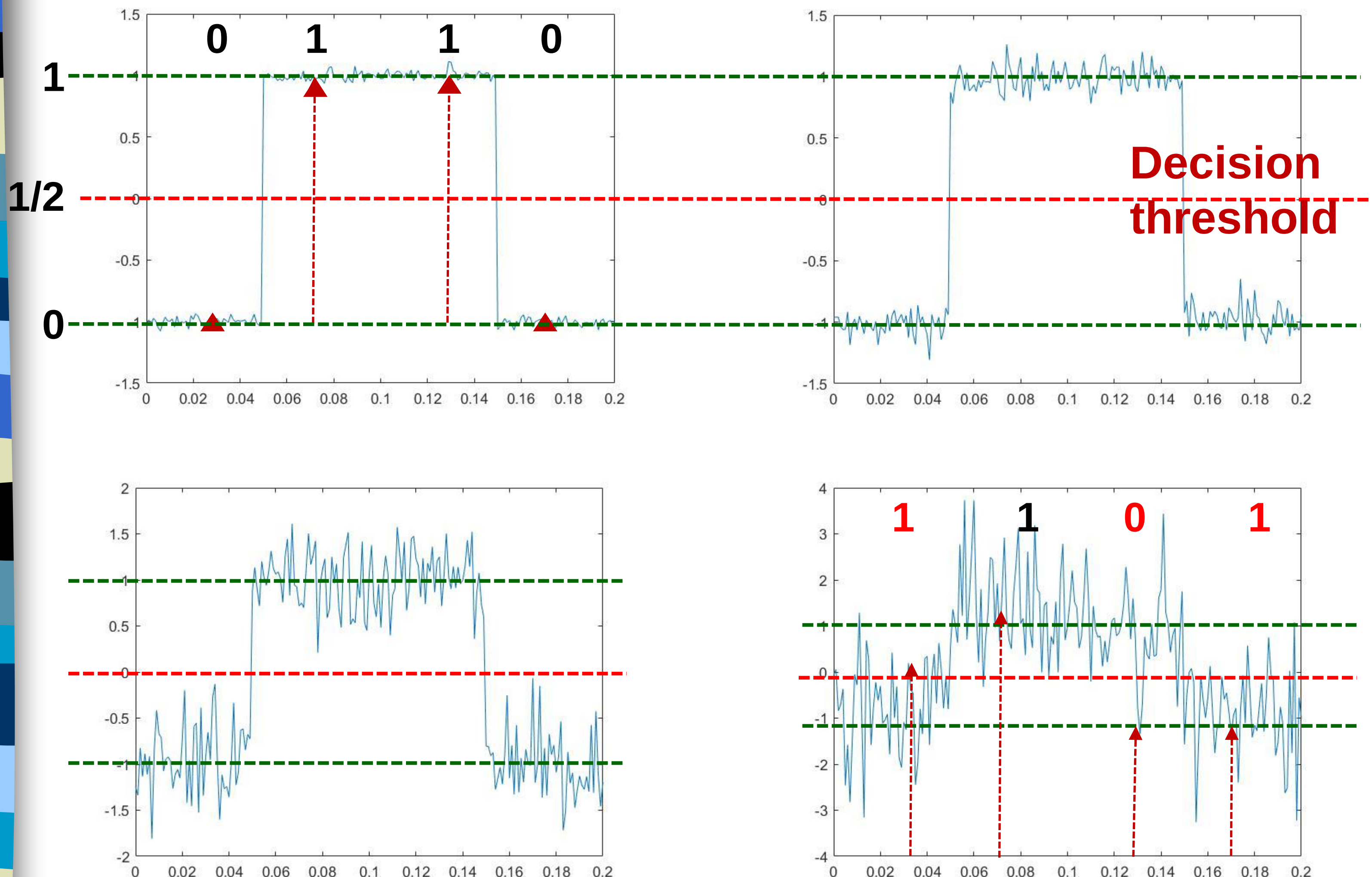
Main concept:

Line code may get deformed to any shape due to ISI + AWGN as far as a receiver takes correct decisions on logical symbols sent.

Decision based approach to detection of a line code:

- **Receiver has to be synchronized with a transmitter**
- **Incoming signal (line code + AWGN + ISI) is sampled**
- **Threshold device makes a decision on symbol sent (0 or 1)**
- **Detected symbols may be formatted to a line code back**

Deformation of a line code by AWGN

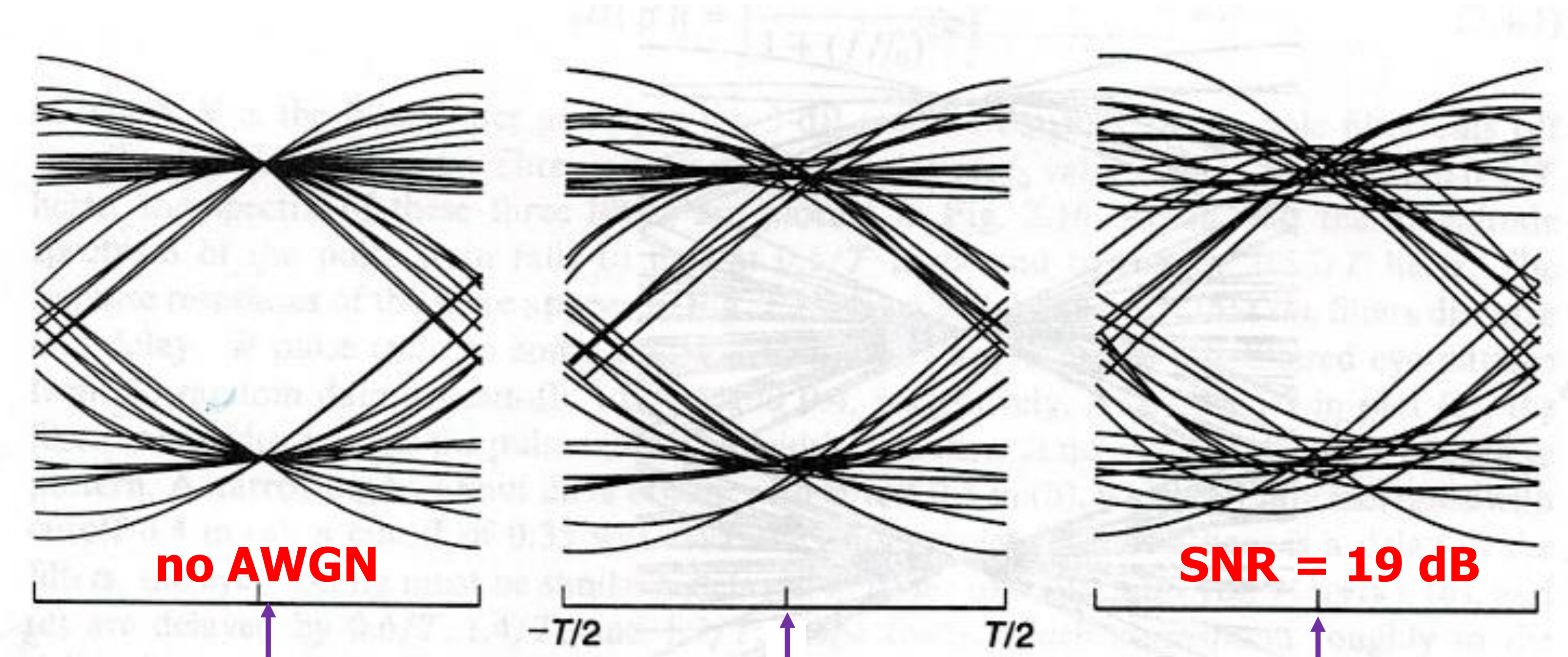


Example: for a moderate SNR = 10 dB error probability ≈ 0.001.
Conclusion: deformation by AWGN may be neglected as compared to ISI.

Deformation of a line code by AWGN (eye diagram)

time base triggered by line code clock

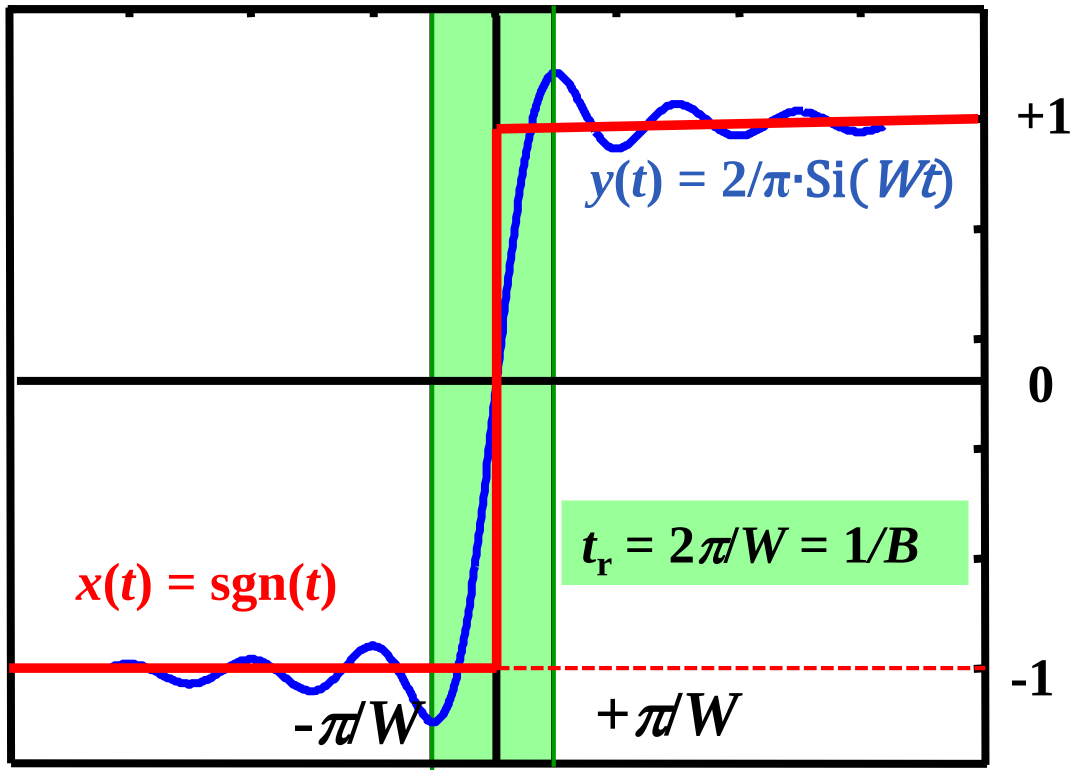
deformed line code by AWGN → **OSCILLOSCOPE** → eye diagram



sampling instants

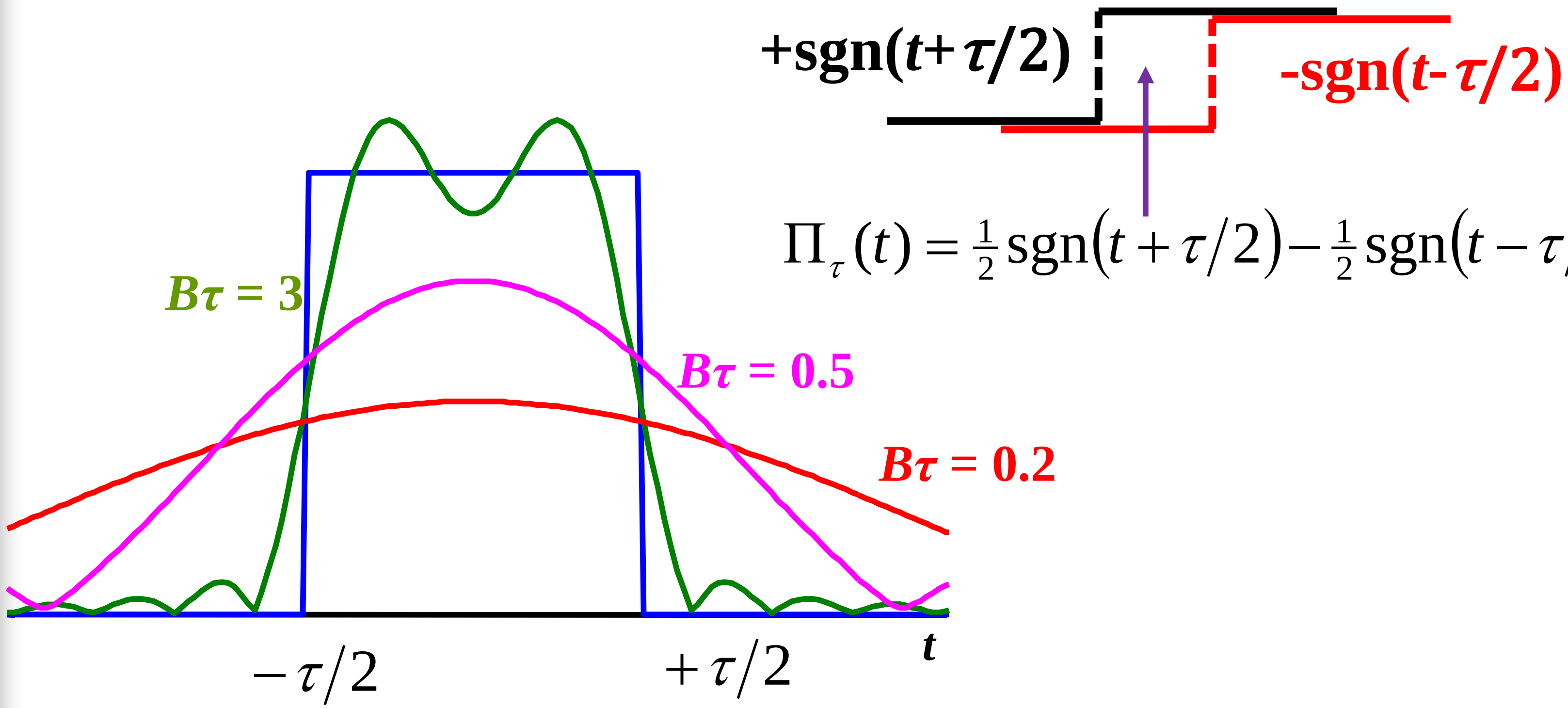
„Digital Transmission Engineering”, J. B. Anderson

Lowpass filtering of sgn(t) (contribution to ISI)



Slope rising time is inveresly proportional to a filter bandwidth.

Lowpass filtering of a rectangular pulse (contribution to ISI)



$$\Pi_\tau(t) = \tfrac{1}{2}\,\mathrm{sgn}\left(t+\tau/2\right) - \tfrac{1}{2}\,\mathrm{sgn}\left(t-\tau/2\right)$$

The narrow a LPF bandwidth is the more its output pulse gets dispersed thereby causing intersymbol interference (ISI) with adjacent pulses. Uncontrolled ISI is a dominant source of errors in digital transmission of signals.

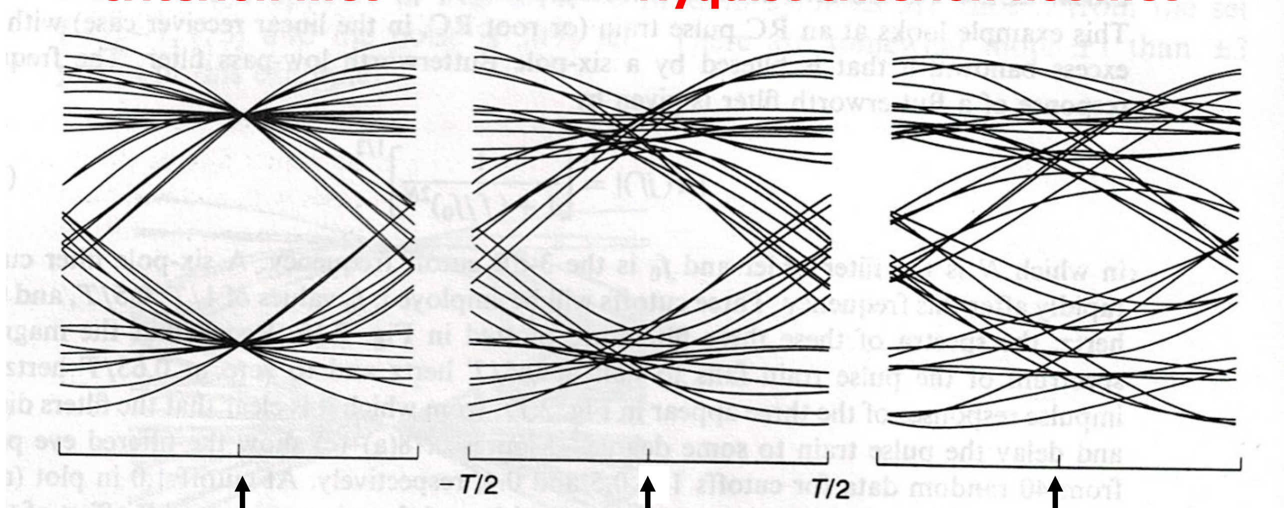
Deformation of a line code by ISI (eye diagram)

time base triggered by line code clock

deformed line code by ISI → OSCILLOSCOPE → eye diagram

ISI = 0, Nyquist criterion met

ISI increasing, Nyquist criterion not met



sampling instants

Trade-off for transmission codes

- Transmission codes are to great extent immune to AWGN as a receiver does not have to recover the exact shape of pulse sent. The single task of the receiver is to take a sample of incoming signal and decide whether „0” or „1” was sent.

- The decision based approach to recover the logical data results in an immunity of a line code transmission to the bandwidth limiting referred to as the InterSymbol Interference (ISI).

- The distortion of a signal shape due to bandlimiting results in overlapping succesive pulses (InterSymbol Interference, ISI) which may effect in decision errors.

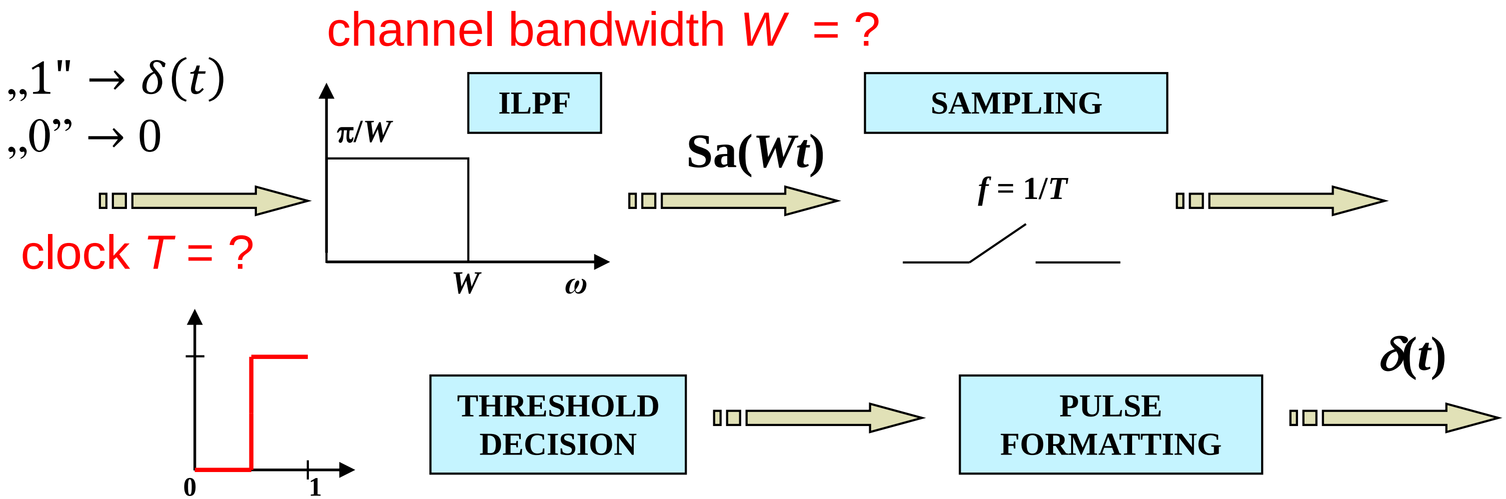
- Intersymbol Interference can be fully eliminated (ISI = 0) provided the Nyquist signaling was applied.

- CONCLUSION: the decision based approach to recover the logical data allows for some deforming of a shape of pulses (used in trans-mission codes) as far as samples taken at resampling instants are correct.

Nyquist signaling

- Rectangular pulse filtering
- Dirac unipolar signaling
- Intersymbol Interference (ISI)
- ISI elimination (Dirac signaling)
- Spectral efficiency of binary signaling
- ISI elimination (non-Dirac signaling)
- ISI elimination (rectangular pulses)
- ISI elimination – Nyquist 1st criterion
- „Raised cosine” shaping
- Summary

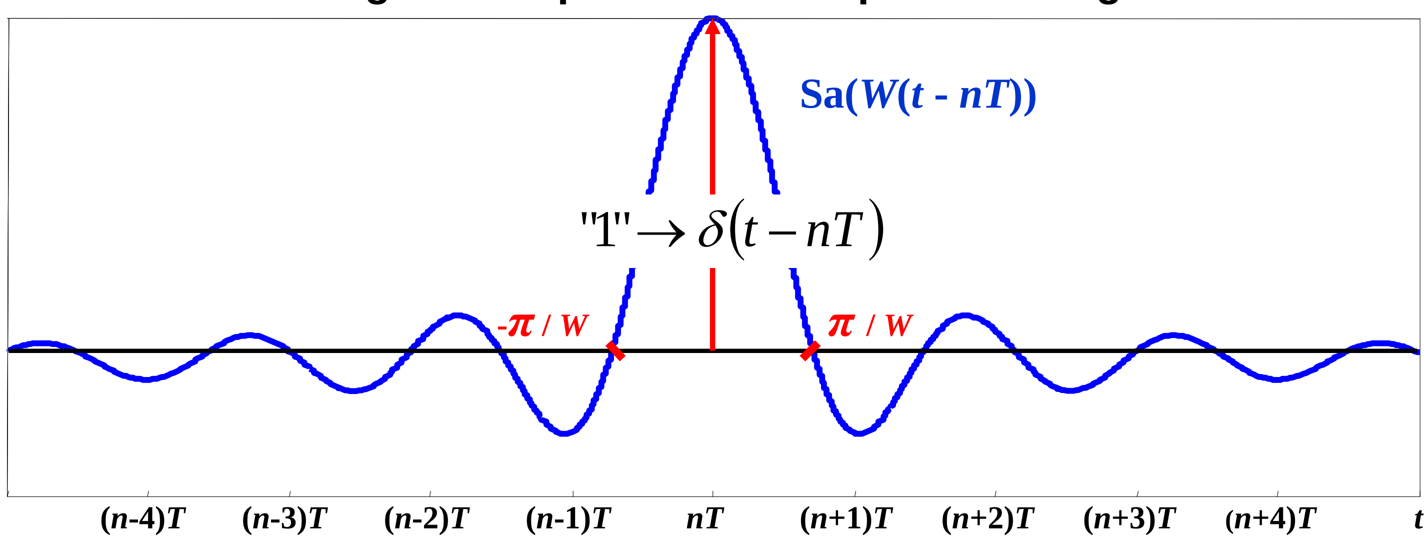
Dirac unipolar signaling



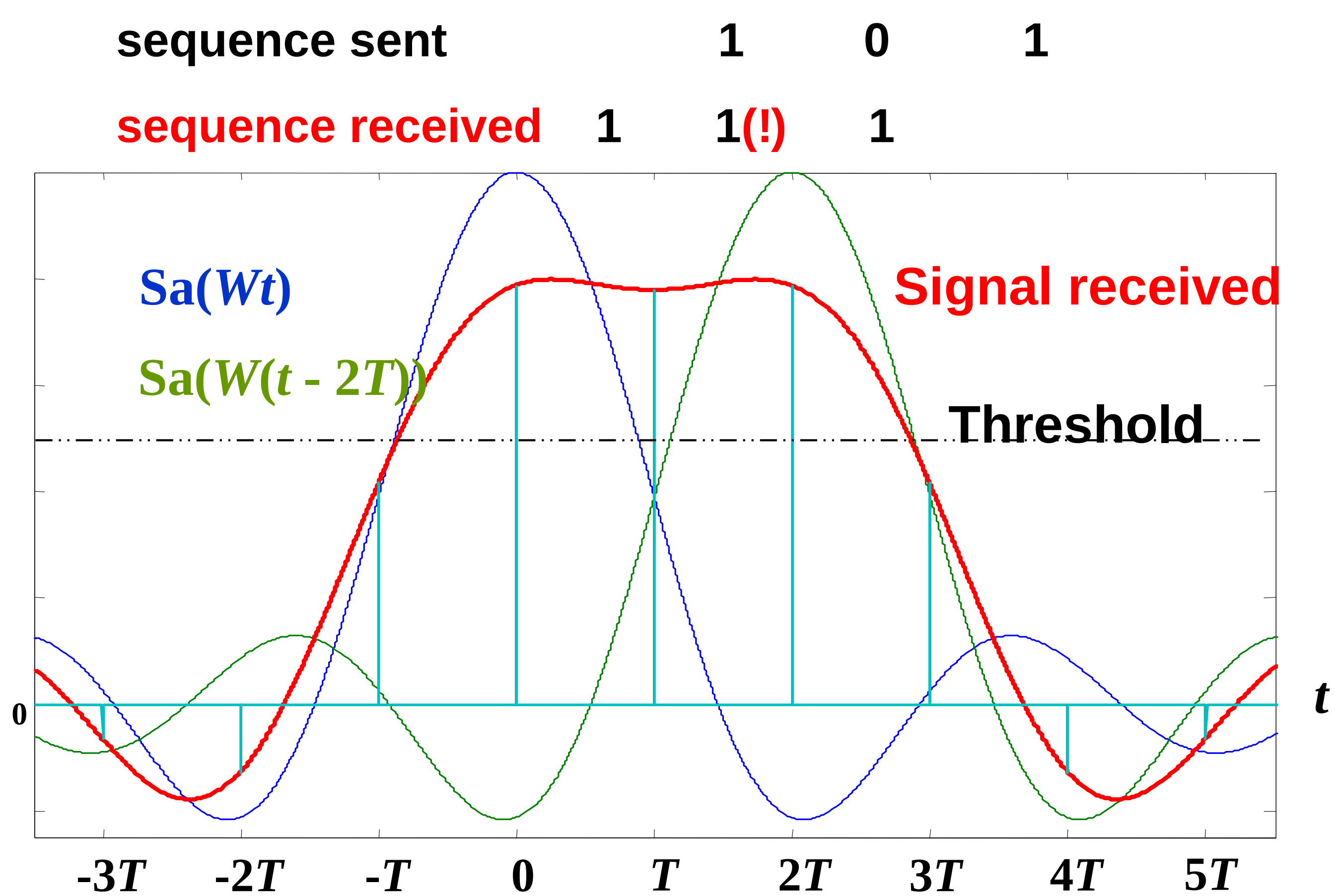
Receiver (RZ signaling with Dirac pulses) – block diagram

(how to tune bandwidth *W* and clock *T* to avoid ISI?)

Single Dirac pulse after lowpass filtering



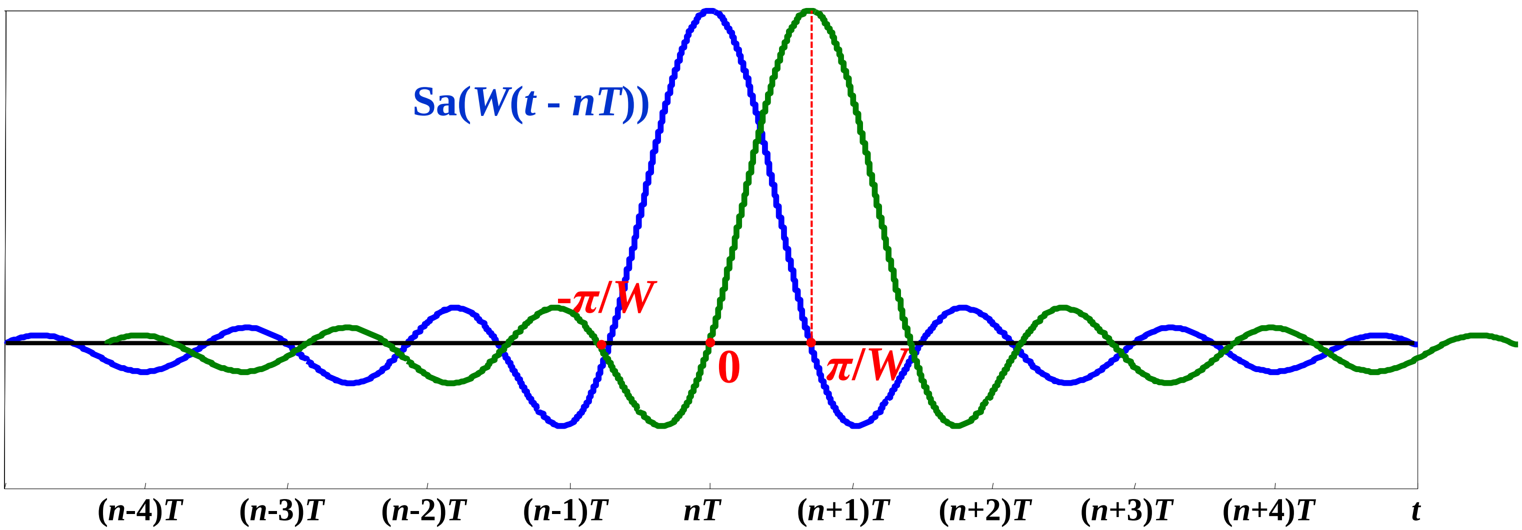
Intersymbol Interference (ISI)



Sequence of three Dirac pulses (1, 0, 1) sent at a rate 1/*T*

ISI elimination (unipolar Dirac signaling)

ISI disappears (ISI = 0) when Dirac pulses are sent at zeroes of the low-pass filter pulse response Sa(*Wt*).

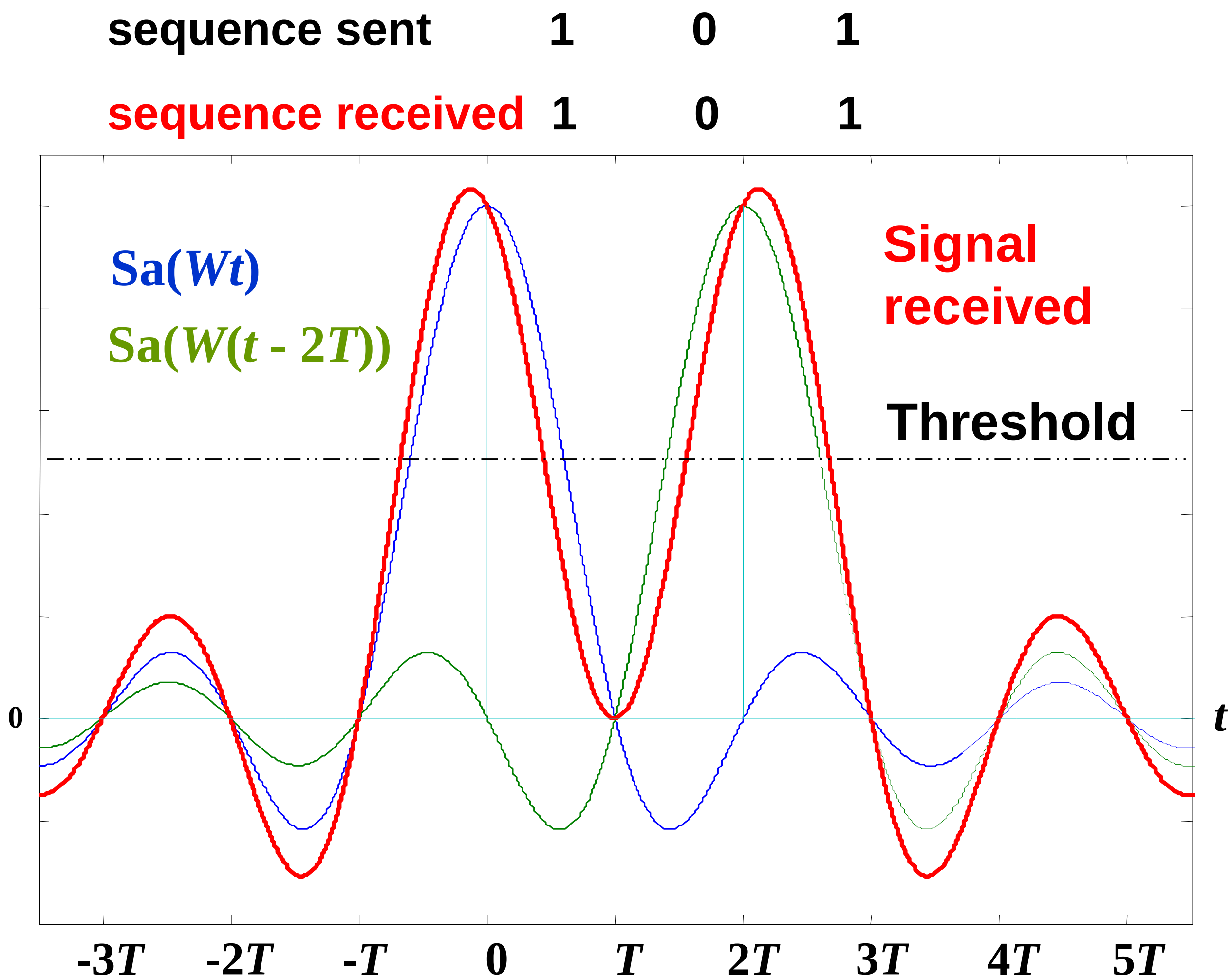


$$T = \frac{\pi}{W} = \frac{1}{2B}$$

$$\text{Nyquist rate:} \quad f_T = 1/T = 2B$$

$$\text{Nyquist interval:} \; T = \frac{1}{2B}$$

ISI elimination (unipolar Dirac signaling)



Sequence of three Dirac pulses (1, 0, 1) sent at the Nyquist rate $1/T = 2B = W/\pi$

Spectral efficiency of binary signaling

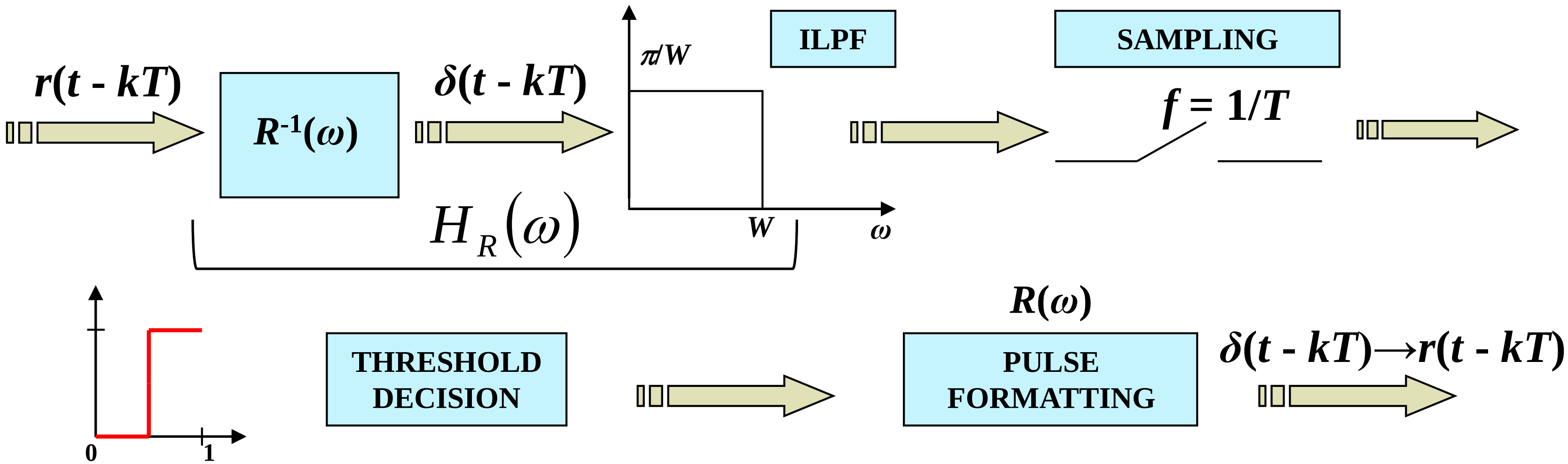
Spectral efficiency R_B of signaling is determined as a signaling rate R achieved from the bandwidth B requested for ISI = 0.

$$R_B[\mathrm{bps/Hz}] = \frac{R[\mathrm{bps}]}{B[\mathrm{Hz}]}$$

Spectral efficiency of the Nyquist baseband signalling

$$R = \frac{1}{T} = 2B \quad \Rightarrow \quad R_B = \frac{R[\mathrm{bps}]}{B[\mathrm{Hz}]} = 2[\mathrm{bps/Hz}]$$

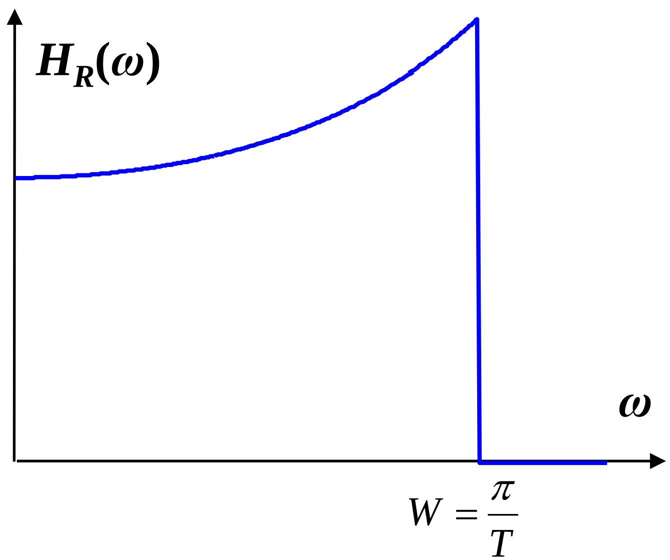
ISI elimination (non-Dirac signaling)



$$H_R(\omega) = \frac{1}{R(\omega)} H(\omega) = \begin{cases} \dfrac{\pi}{WR(\omega)}, & |\omega| \le W \\ 0, & |\omega| \ge W \end{cases}$$

Full ISI elimination at non-Dirac signaling requests a channel frequency characteristics compensation related to a Fourier transform of a pulse used for signaling.

ISI elimination (rectangular pulses)

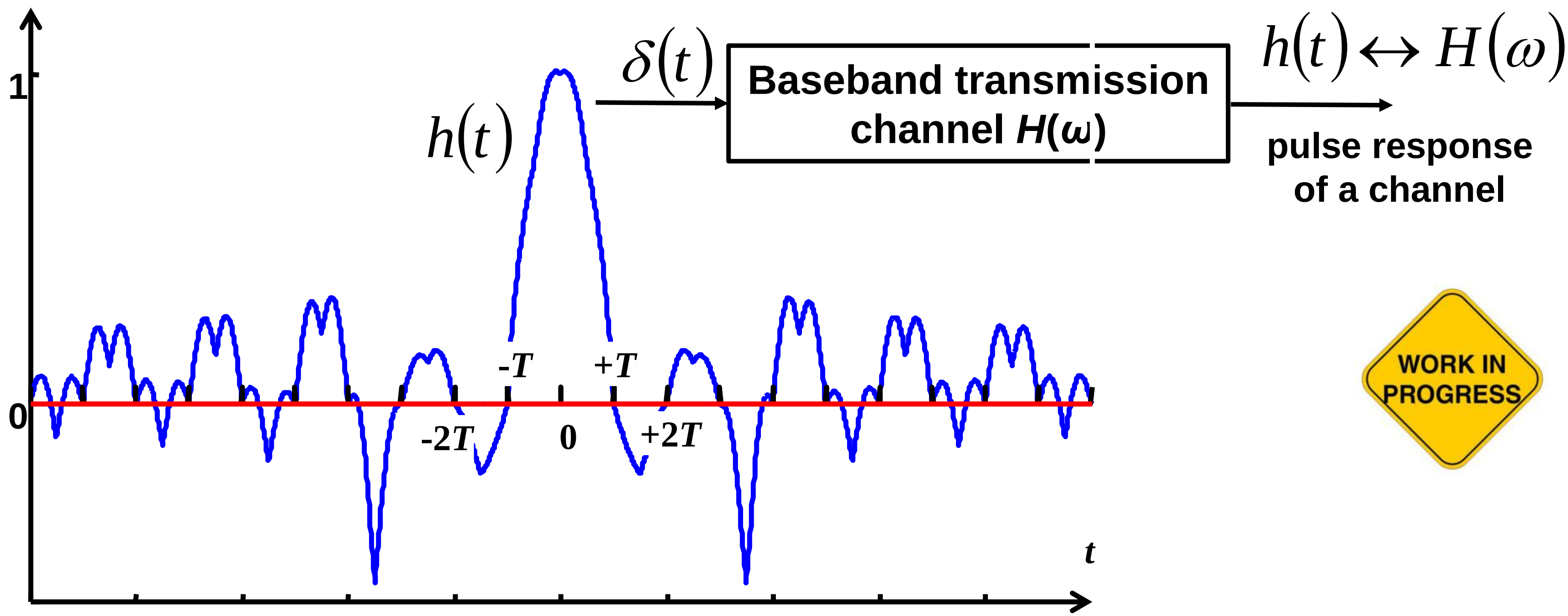


$$r(t) = \Pi_T(t) \leftrightarrow$$

$$T\mathrm{Sa}\,\frac{\omega T}{2} = \frac{\pi}{W}\mathrm{Sa}\,\frac{\pi\omega}{2W}$$

$$H_R(\omega) = \begin{cases} 1\Big/\mathrm{Sa}\,\dfrac{\omega\pi}{2W}, & |\omega| \le W \\ 0, & |\omega| \ge W \end{cases}$$

IMS elimination – Nyquist filter

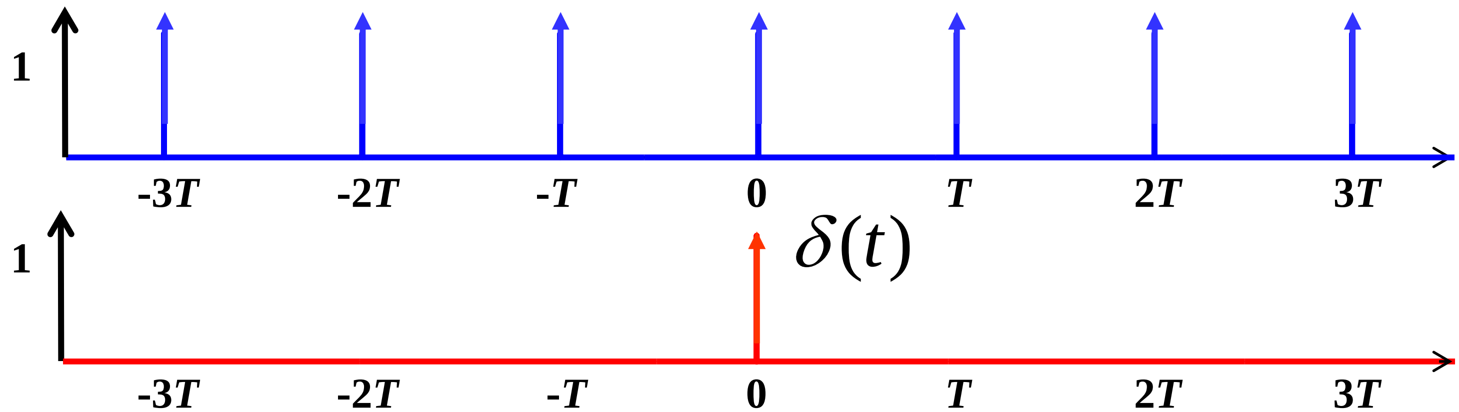


So far we were considering a „brickwall" shaped baseband transmission channel resulting in ISI = 0. Question – are there other transmission channels providing ISI = 0?

We are looking for a channel impulse response $h(t)$ having regular zeroes $h(t = kT) = 0$, $k = \pm 1, \pm 2\ldots$ at T [units of time] apart except the central zero $h(t = 0) = 1$. Such a channel follows the Nyquist 1st criterion ensuring ISI = 0.

IMS elimination – Nyquist filter

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



Signal ***h*(*t*)** matching the 1st Nyquist criterion has to break all theeth of a Dirac comb out except the central tooth.

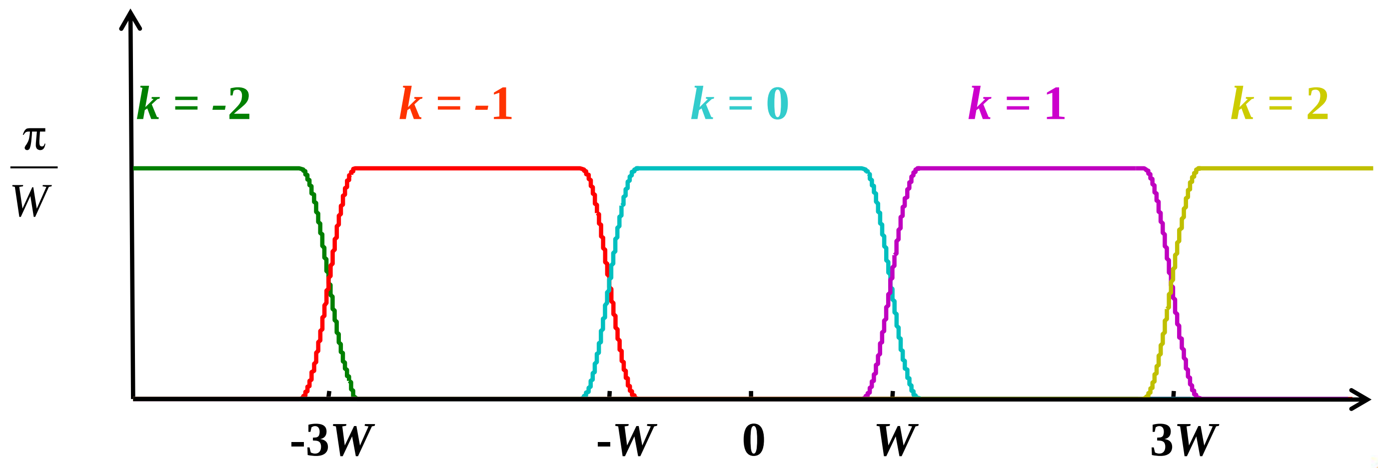
$$h(t)\delta_T(t) = \delta(t)$$

IMS elimination – Nyquist filter

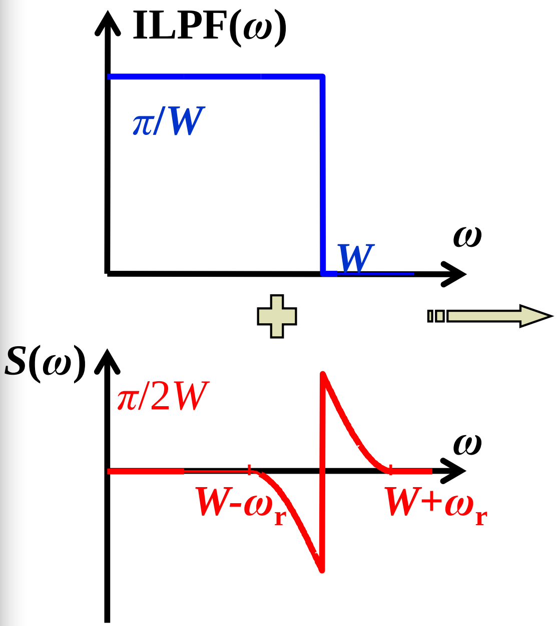
$$\sum_{k=-\infty}^{\infty} H(\omega - k2W) = T = \frac{\pi}{W}$$

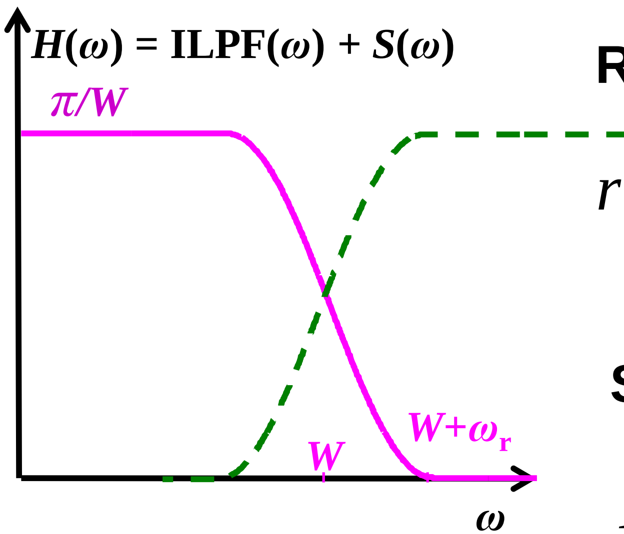
$$T = \frac{\pi}{W} = \frac{1}{2B}$$

Nyquist rate



IMS elimination – Nyquist filter





Roll-off factor:

$$r = \frac{\omega_r}{W} \qquad 0 \le r \le 1$$

Spectral efficiency:

$$R_B = \frac{2}{1+r}$$

Smooth slope of a filter passband →
→ bandwidth extension (*W* → *W*$_+$)

$$W_+ = W + \omega_r = W + rW = (1+r)W$$

$$h(t) = \mathrm{Sa}(Wt) + s(t) = \mathrm{Sa}(Wt) + S \times \frac{2}{W} \sin(Wt)$$

S **– constant depends on *S*(*ω*)**

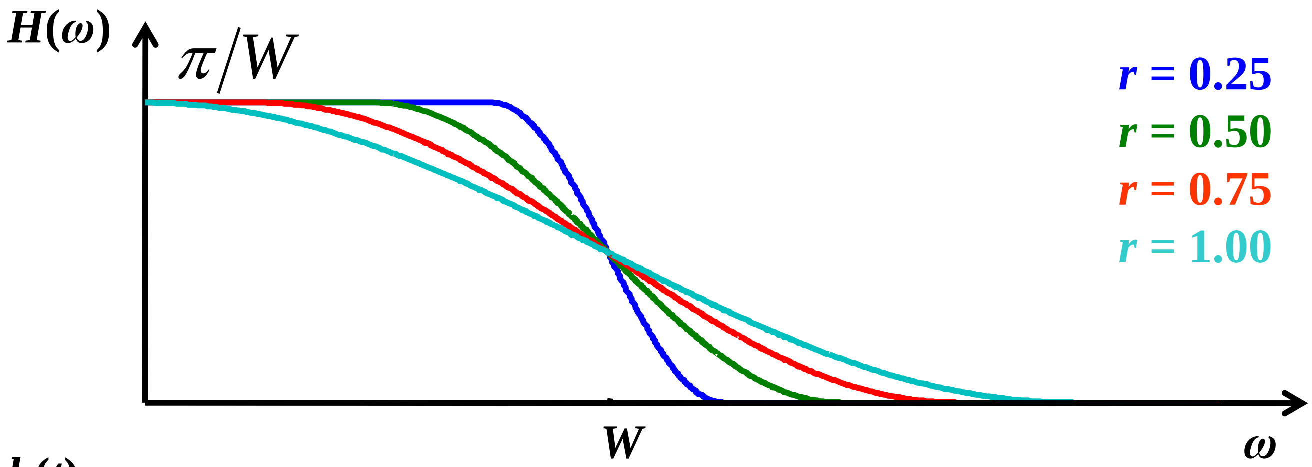
„Raised cosine” Nyquist shaping

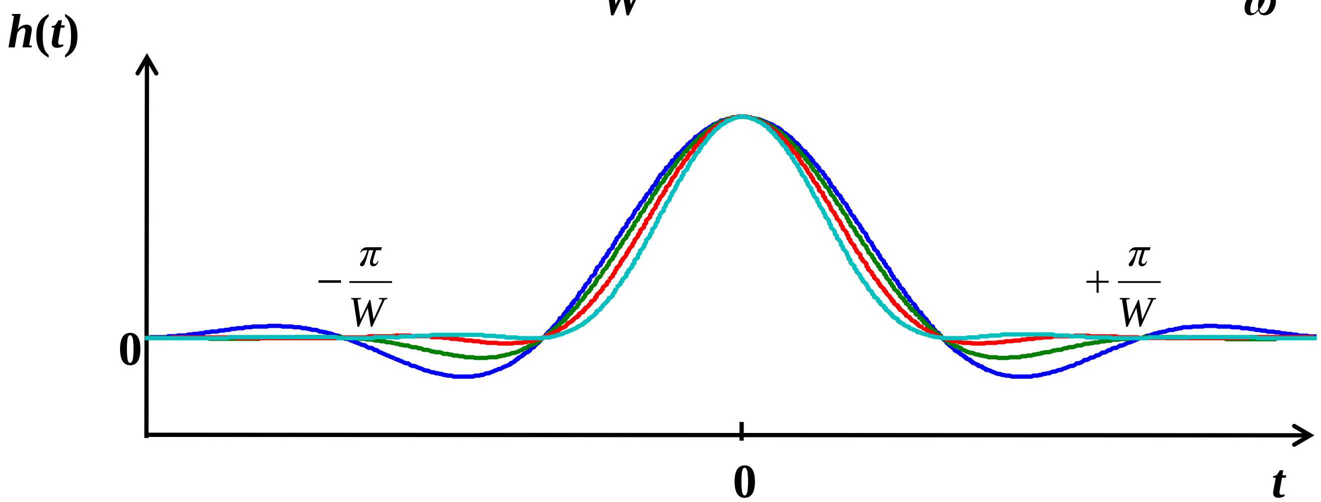
$$H(\omega)=\begin{cases}\dfrac{\pi}{W}, & |\omega|<W-\omega_{\mathrm{r}}\\ \dfrac{\pi}{2W}\left(1-\sin\pi\dfrac{\omega-W}{2\omega_{\mathrm{r}}}\right) & W-\omega_{\mathrm{r}}\le|\omega|\le W+\omega_{\mathrm{r}}\\ 0 & |\omega|>W+\omega_{\mathrm{r}}\end{cases}$$

Pulse response of a „raised cosine” shaped filter

$$h(t)=\mathrm{Sa}(Wt)\frac{\cos\omega_{\mathrm{r}}t}{1-\left(2\omega_{\mathrm{r}}t/\pi\right)^{2}}$$

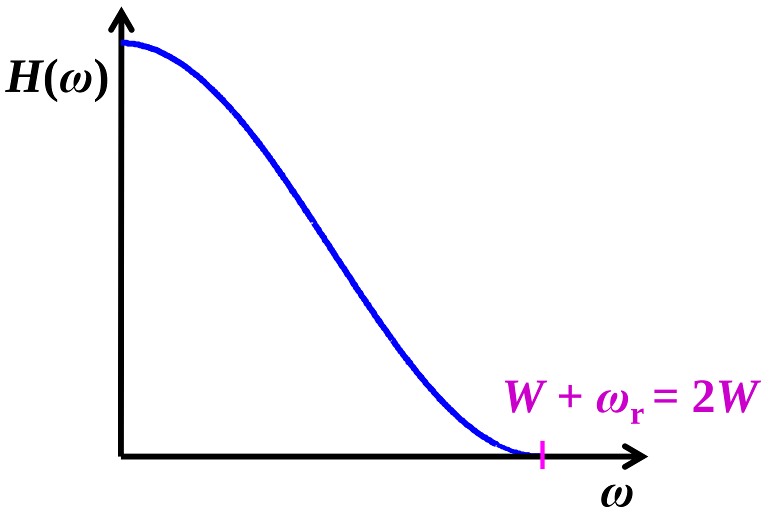
„Raised cosine" Nyquist shaping

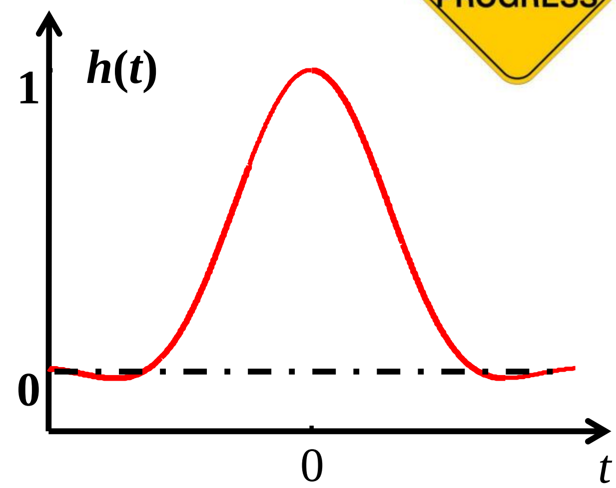




The smoother filter edge is the longer the impulse response is.

„Raised cosine” shaping – special case





For $\omega_r = W$ **(*r* = 1)**

$$H(\omega) = \begin{cases} \dfrac{\pi}{2W}\left(1 + \cos\dfrac{\pi\omega}{2W}\right), & |\omega| \le 2W \\ 0, & |\omega| \ge 2W \end{cases} \quad \leftrightarrow \quad h(t) = \mathrm{Sa}\,Wt\,\frac{\cos Wt}{1 - \left(\dfrac{2Wt}{\pi}\right)^2}$$

Square root raised cosine pulse

It may be proved that the optimum transmission under superposition of AWGN and ISI = 0 needs splitting the „raised cosine” transfer function into transmitter „square root raised cosine” transfer function and a similar receiver „square root raised cosine” transfer function *g*(*t*) ↔ *G*(*ω*). The entire channel transfer function equals to „raised cosine” .

$$G(\omega) = \sqrt{H(\omega)} = ?$$

$$G(\omega) \leftrightarrow g(t) = ?$$

$$H(\omega) = \begin{cases} \dfrac{\pi}{2W}\left(1 + \cos\dfrac{\pi\omega}{2W}\right), & |\omega| \le 2W \\ 0, & |\omega| \ge 2W \end{cases}$$

- **Draw plots of a „raised cosine” and a „square root raised cosine”**
- **Draw spectra of a „raised cosine” and „square root raised cosine”**
- **Check whether a „square root raised cosine” provides ISI = 0 by itself**

Summary

- **Noise/error sources**
  - **Channel noise** – low impact in a baseband wired transmission, SNR high enough
  - **Quantization noise** – to be greatly reduced with increasing # quantization levels resulting in a transmission rate growth (that in turn is reduced by compression techniques)
  - **Intersymbol Interference** – real source of errors to be fight against 1.Nyquist signaling, 2. Nyquist shaping, 3. Channel equalization

- **1st Nyquist criterion**
  - **ISI = 0** when a signaling clock follows the **Nyquist clock.**
  - **Nyquist filter slope shaping** allows for deploying **channels having a smooth amplitude characteristics** at the cost of **decreasing the spectral efficiency** of signaling.
  - **Raised cosine** (raised cosine) and **square root raised cosine** are the most common Nyquist shapings.
  - **Spectral efficiency of signaling** is measured as a ratio of a transmission rate to a bandwidth requested for supporting this rate (likely for ISI = 0).